Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY

Winter Examination-2021

Subject Name: Mathematical Methods - I

Subject Code: 5SC03MAM1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 14/12/2021** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

(a) State second shifting theorem for Laplace transform. 02

(b) $$L^{-1}\left\{\frac{1}{s^{\frac{7}{2}}}\right\} = \_\_\_\_.$$ 02

(c) Define: Error function. 01

(d) $$L\left\{\frac{f(t)}{t}\right\} = \_\_\_\_\_.$$ 01

(e) Find $Z\left(\frac{1}{n!}\right)$. 01

Q-2 **Attempt all questions** **(14)**

(a) If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, evaluate u_2 and u_3. 05

(b) If $L\{f(t)\} = \overline{f}\,(s)$ then prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{dx^n}\left[\overline{f}\,(s)\right]$. 05

(c) Find $L\{cos2t + t^2 \sin at + 2\sin^2 t\}$. 04

OR

Q-2 **Attempt all questions** **(14)**

(a) Find the Z transform and region of convergence of 05

$$u(n) = \begin{cases} 4^n & for\ n < 0 \\ 2^n & for\ n \geq 0 \end{cases}.$$

(b) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\sin x\; y(0) = 0, y'(0) = 1$ by using Laplace transform. 05

(c) Prove that $L\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$. 04

Q-3		**Attempt all questions**	**(14)**
	(a)	Evaluate: $L^{-1}\left\{\frac{e^{4-3s}}{(s+4)^{\frac{5}{2}}}\right\}$	05
	(b)	Find $Z^{-1}\left\{\frac{2(z^2-5z+6.5)}{(z-2)(z-3)^2}\right\}, 2<\lvert z\rvert<3.$	05
	(c)	If $f(t)$ is periodic function with period T then prove that $$L\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt.$$	04

OR

Q-3		**Attempt the Following questions**	**(14)**
	(a)	State and prove Convolution theorem for Laplace transform.	05
	(b)	Evaluate: $\int_0^\infty e^{-t}\left(\int_0^t u^2 \sinh u \cosh u \; du\right)dt.$	05
	(c)	If $\{u_n\}$ be any discrete sequence and $Z\{u_n\}=U(z)$ then prove that $(i) Z(a^{-n}u_n)=U(az)$ and $(ii)\; Z(a^n u_n)=U\left(\frac{z}{a}\right).$	04

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	(a)	State Dirichlet's condition for Fourier series.	02
	(b)	Check whether the function $f(x)=\begin{cases}0\,; & -2<x<-1\\ k; & -1<x<1\\ 0\,; & 1<x<2\end{cases}$ is even or odd?	02
	(c)	Show that $\mathcal{F}[xf(x)]=i\frac{d}{d\lambda}\left(F(\lambda)\right)$	02
	(d)	Define finite Fourier sine transform.	01

Q-5		**Attempt all questions**	**(14)**
	(a)	Find the complex Fourier series of $f(x)=e^{-x}, -\pi<x<\pi$ and $f(x+2\pi)$.	05
	(b)	State and prove Parseval's formula for Fourier series.	05
	(c)	Find Fourier sine series of period 4 for the function $f(x)=\begin{cases}2x & ; 0<x<1\\ 4-2x & ; 1<x<2\end{cases}.$	04

OR

Q-5 **Attempt all questions** **(14)**

(a) Find the Fourier series of $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & ; -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & ; \quad 0 < x < \pi \end{cases}$ and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots\cdots = \frac{\pi^2}{8}$. 06

(b) Find Fourier cosine transform of e^{-x^2}. 05

(c) If $F(\lambda)$ is Fourier transform of $f(x)$ then prove that 03

$$\mathcal{F}[f(ax)] = \frac{1}{a} F\left(\frac{\lambda}{a}\right), a \neq 0.$$

Q-6 **Attempt all questions** **(14)**

(a) Solve integral equation $\int_0^\infty f(v) \cos \lambda v \, dv = \begin{cases} 1 - \lambda \,; 0 \leq \lambda \leq 1 \\ 0 \quad ; \quad \lambda > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. 07

(b) Find Fourier transform of $f(x) = e^{-ax^2}; a > 0$ and hence deduce that $F\left(e^{-\frac{x^2}{2}}\right) = e^{-\left(\frac{\lambda^2}{2}\right)}$. 07

OR

Q-6 **Attempt all Questions** **(14)**

(a) Find the temperature $u(x,t)$ in a slab whose ends $x = 0$ and $x = a$ are kept at temperature zero and whose initial temperature is $\sin\left(\frac{\pi x}{a}\right)$. 09

(b) Express $e^{-x} \cos x$ as a Fourier cosine integral and show that $e^{-x} \cos x = \frac{2}{\pi} \int_0^\infty \frac{(\lambda^2+2)}{\lambda^4+4} \cos \lambda x \, d\lambda$. 05